Effective Homology and Spectral Sequences

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ABSTRACT

Effective homology and spectral sequences are two different techniques of Algebraic Topology which can be used for the computation of homology and homotopy groups. In this work we try to relate both methods, showing that the effective homology method can also be used to produce algorithms computing some spectral sequences. In the thesis we focus our attention on two particular situations: spectral sequences associated with filtered complexes and the Bousfield-Kan spectral sequence.

The first part of the memoir is devoted to spectral sequences of filtered complexes, which under some good conditions converge to the homology groups of the initial complex. Making use of the effective homology method, we have developed several algorithms computing the different components of these spectral sequences: groups and differential maps in every stage, convergence level and filtration of the homology groups induced by the initial filtration. These algorithms have been implemented as a new module for the Kenzo system, a program of Symbolic Computation in Algebraic Topology, and can be used to determine two classical examples of spectral sequences, those of Serre and Eilenberg-Moore.

Other spectral sequences are not defined by means of filtered complexes. This is the case of the Bousfield-Kan spectral sequence, related with the computation of homotopy groups, which is studied in the second part of the thesis. Our algorithms for the computation of spectral sequences of filtered complexes cannot be applied, but the effective homology method can be useful again to develop a constructive version of the Bousfield-Kan spectral sequence. As a first necessary step, our main result is an algorithm computing the effective homology of the free simplicial Abelian group RX generated by a 1-reduced simplicial set X. This algorithm allows the construction of the levels 1 and 2 of the spectral sequence; for the computation of the higher “pages”, we present a sketch of a new algorithm which is not finished yet. Furthermore, we include a proof of the convergence of the spectral sequence.